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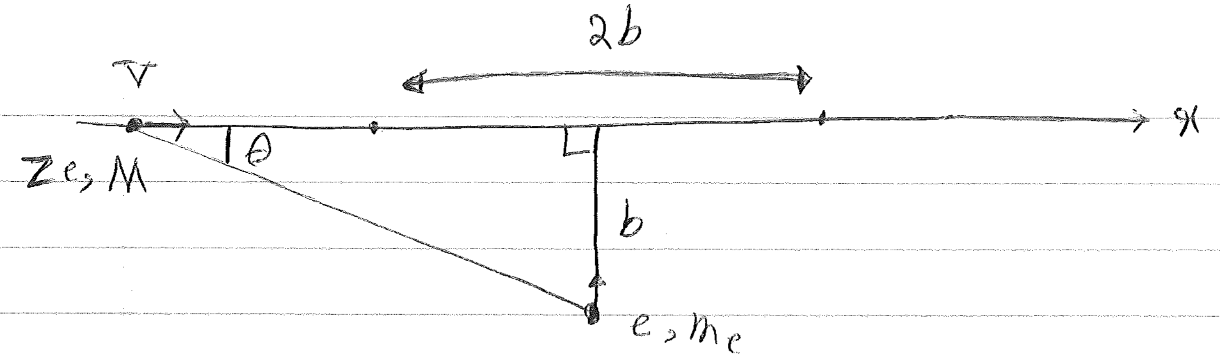
Interaction of High Energy Particles: Ionization Loss

We will consider various processes between high-energy particles and matter. These processes provide different mechanisms for emission of photons from charged particles, as well as energy loss mechanisms for high-energy photons and charged particles.

The first type of interaction we are going to consider is ionization. In the process of ionization, electrons are torn off atoms by the electrostatic force between the charged high-energy particle and the electrons. This process is important for many reasons. First of all, ionization caused by high-energy particles in detectors can be used to measure the flux of particles and some of their other properties. Second, the process influences the propagation of high-energy particles under cosmic conditions. Third, these losses provide an effective mechanism for heating the interstellar gas, in particular the cool giant molecular clouds in galaxies.

Now let us look at some general features of the physical process. We consider first the collision of high energy protons and nuclei with stationary electrons. In this case only a very small fraction of the high-energy particle is transferred to the electron, and hence it will move undeflected, virtually at the same velocity, throughout the process. The interaction is illustrated in the figure next page. Stationarity of the electron assumes that its orbital motion is frozen within the characteristic time scale for its interaction with the



high-energy particle. Moreover, it will not significantly move within the duration of collision as a result of momentum transfer. We will find the constraints that these assumptions impose later on. Finally, we consider non-relativistic motion and work in the classical limit. The full result from relativistic quantum theory will be presented later.

The total momentum impulse given to the electron is $\int \vec{F}\, dt$. $\vec{F}$ is the Coulomb force between the particles and, by symmetry, the forces parallel to the line of flight of the high-energy particle cancel out. Then:

$$F_\perp = \frac{Ze^2}{r^2} \sin\theta \quad , \quad dt = \frac{dx}{v}$$

↑(we use Gaussian units)

$$\int_{-\infty}^{+\infty} F_\perp \, dt = \int_0^{\pi} \frac{Ze^2}{b^2} \sin^2\theta \frac{b \sin\theta}{v \sin^2\theta} \, d\theta = -\frac{Ze^2}{bv} \int_0^{\pi} \sin\theta \, d\theta$$

The momentum impulse therefore is:

$$p = \frac{2Ze^2}{bv}$$

The kinetic energy transferred to the electron is found to be:

$$\frac{p^2}{2m_e} = \frac{2Z^2e^4}{b^2v^2m_e}$$

We now want the average ionization energy loss per unit length as the high-energy particle traverses the medium. Assuming a uniform density of electrons n_e, the number of collisions with collision parameters in the range b to $b+db$ can be found and integrated over b. The total energy loss $-dE$ in the length dx is:

$$\left[\int_{b_{min}}^{b_{max}} n_e \, 2\pi b \, db \, \frac{2Z^2e^4}{b^2v^2m_e}\right] dx$$

Thus:

$$\frac{dE}{dx} = \frac{-4 Z^2 e^4 n_e}{v^2 m_e} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

Here b_{min} and b_{max} denote the minimum and maximum values of the collision parameter, respectively, for which the assumption of electron stationarity is valid.

b_{max} can be estimated as follows. The interaction is the strongest when the high energy particle is at the distance of closest approach to the electron. This results in an estimate for the characteristic time scale $\tau \simeq \frac{2b}{v}$ (see the figure on the previous pages). This needs to be much shorter than the period for orbital motion of the electrons which is $\frac{2\pi}{\omega_0}$ (ω_0 being the angular frequency for the orbital motion). This yields:

$$b_{max} \approx \frac{\pi v}{\omega_0}$$

We can also find an estimate of b_{min}. The electron also moves as a result of the transferred momentum. The distance traveled within the time τ in this way must be much smaller than b. This results in:

$$\frac{P}{2m_e} \times \frac{2b}{v} \ll b \Rightarrow b_{min} \approx \frac{2Ze^2}{m_e v^2}$$

Putting things together, we find:

$$\frac{dE}{dx} = -\frac{4Z^2 e^4 n_e}{v^2 m_e} \ln\left(\frac{\pi m_e v^2}{2Ze^2\omega_0}\right)$$

The orbital angular frequency can be related to the ionization energy I. In practice, we should use properly weighted mean over all states of the electrons in the atom. Simple modification of the above expression, which uses the deBroglie wavelength of the electron for

b_{min}, then results in:

$$\frac{dE}{dx} = -\frac{4 Z^2 e^4 n_e}{v^2 m_e} \ln\left(\frac{m_e v^2}{2 \bar{I}}\right)$$

Here $\bar{I}$ is the properly weighted mean of I. We note that the value of $\bar{I}$ cannot be calculated exactly except for the simplest atoms and has to be found by experiment.

An important point is the inevitability of using quantum mechanics to find the ionization energy loss rate. The classical treatment in above only makes sense if the de Broglie wavelength of the electron, denoted by λ_e, is much smaller than the collision parameter b. This requires that:

$$\lambda_e \simeq \frac{\hbar}{p} << b \Rightarrow \frac{\hbar b v}{2Ze^2} << b \Rightarrow \frac{v}{c} << \frac{2Ze^2}{\hbar c} \Rightarrow$$

$$\frac{v}{c} \lesssim 0.01\, Z \quad \left(\alpha \equiv \frac{e^2}{\hbar c} = \frac{1}{137} \text{ ; fine structure constant}\right)$$

This implies that quantum effects become important before relativistic effects.

The exact result derived from relativistic quantum theory is known as the Bethe-Bloch formula:

$$\frac{dE}{dx} = -\frac{4 Z^2 e^4 n_e}{m_e v^2}\left[\ln\left(\frac{2\gamma^2 m_e v^2}{I}\right) - \frac{v^2}{c^2}\right]$$

An interesting aspect of this formula is that in the non-relativistic limit ($v \ll c$) we have $\frac{dE}{dx} \propto E^{-1}$.

However, in the relativistic regime ($v \simeq c$) we have $\frac{dE}{dx} \simeq$ Const. (except for logarithmic dependence on E).

We also note that $\frac{dE}{dx}$ does not depend on the mass of the high-energy particle.

We conclude by briefly describing the situation when the high-energy particle itself is an electron.

In this case, the high-energy particle will suffer large deviations as it has the same mass as the "target" electron. The net result, however, is not very different from the Bethe-Bloch formula. The expression for the energy loss of electrons is:

$$\frac{dE}{dx} = \frac{2e^4 n_e}{m_e v^2}\left[\ln\left(\frac{\gamma^2 m_e v^2 E_{max}}{2I^2}\right) - \left(\frac{2}{\gamma} - \frac{1}{\gamma^2}\right) + \frac{1}{\gamma^2} + \frac{1}{8}\left(1 - \frac{1}{\gamma}\right)^2\right]$$

Here E_{max} is the maximum kinetic energy which can be transferred to the "target" electron. It can be shown that it has the value:

$$E_{max} = \frac{\gamma^2 m_e v^2}{1+\gamma}$$